ONLINE SUPPLEMENT

Song, Xi and Robert D. Mare. 2015. “Retrospective Versus Prospective Approaches to the Study of Intergenerational Social Mobility.” Sociological Methods and Research 44(4): 555-584.

Appendix A: Simulation Details

This appendix provides the details for the simulation examples. For the two-generation model, we assume that a man’s fertility depends on his own socioeconomic position, and his socioeconomic position depends on only his father’s position. There is no lagged effect from the grandfather in both the fertility and mobility equations. The data are generated in the following order according to the specified probability models:

1.1 The exogenous variable $U_1$ for the fathers’ generation is drawn from a standard normal distribution $U_1 \sim N(0, 1)$.

1.2. We then generate the father’s position $Y_1$ (1 vs. 2) for each of the 10,000 subjects:

$$\text{logit}(P[Y_1 = 2|U_1]) = \alpha_0 + \alpha_1 \cdot U_1 = \log\left(\frac{0.3}{0.7}\right) + \log(2) \cdot U_1$$  (A-1)

1.3. The conditional distribution of a father’s fertility given $Y_1$ follows a Poisson distribution with the mean of the fertility satisfies the equation below.

$$E(F_1|Y_1) = \exp(\beta_0 + \beta_1 \cdot (Y_1 - \bar{Y}_1)) = \exp(\log(1.1) + 0.6 \cdot (Y_1 - \bar{Y}_1))$$  (A-2)

We then generate a dichotomous variable $D_1$ based on $F_1$, where $D_1 = 1$ if $F_1 > 0$ and $D_1 = 0$ if $F_1 = 0$.

1.4. The conditional distribution of a son’s variable $U_2$ given $U_1$ and $D_1$ is drawn from a normal distribution, where the standard deviation is fixed at 1 and the mean satisfies the equation below.

$$E(U_2|U_1, D_1 = 1) = \gamma_1 \cdot (U_1 - \bar{U}_1) = 0.8 \cdot (U_1 - \bar{U}_1)$$  (A-3)
1.5. The conditional distribution of a son’s socioeconomic position $Y_2$ given $U_2, D_1$ and $Y_1$ follows a Bernoulli distribution.

$$\logit(P[Y_2 = 2|U_2, Y_1, D_1 = 1]) = \delta_0 + \delta_1 \cdot U_2 + \delta_2 \cdot Y_1$$

(A-4)

$$= \log\left(\frac{0.2}{0.8}\right) + \log(2) \cdot U_2 + \log(2.5) \cdot Y_1$$

1.6. The conditional distribution of a son’s fertility $F_2$ given $Y_2$, and $D_1$ follows a Poisson distribution with the mean of the fertility satisfies the equation below.

$$E(F_2|Y_2, D_1 = 1) = \exp(\beta_0 + \beta_1 \cdot (Y_2 - \bar{Y})_2) = \exp(\log(1.1) + 0.6 \cdot (Y_2 - \bar{Y}))$$

(A-5)

We then generate a dichotomous variable $D_2$ based on $F_2$, where $D_2 = 1$ if $F_2 > 0$ and $D_2 = 0$ if $F_2 = 0$.

In the prospective sample, all the variables $F_1, F_2, D_1, D_2, Y_1, Y_2$ are observed, while in the retrospective sample, we only know $F_i > 0$ (i.e., $D_i = 1$), $F_2, D_2, Y_1, Y_2$. We need to use the proportion of childless adults in the sons’ generation ($D_2 = 1$) to approximate that of the fathers’ generation ($D_1 = 1$) in the adjusted retrospective method.

For the three-generation model, we assume that a man’s fertility depends on the socioeconomic positions and fertility of all prior generations, as well as his own socioeconomic position. In addition, we assume a man’s socioeconomic position depends on the socioeconomic positions of all prior generations. We generate the data by the following steps.

2.1. The exogenous variable $U_1$ for the grandfathers’ generation follows a standard normal distribution $U_1 \sim N(0, 1)$.

2.2. The conditional distribution of a grandfather’s socioeconomic position $Y_1$ (1 vs. 2) given $U_1$, follows a Bernoulli distribution.
2.3. The conditional distribution of a grandfather’s fertility $F_1$ given $Y_1$ follows a Poisson distribution with the mean of the fertility satisfies the equation below.

$$E(F_1 | Y_1) = \exp(\beta_0 + \beta_1 \cdot (Y_1 - \bar{Y})) = \exp(\log(1.1) + 0.6 \cdot (Y_1 - \bar{Y}))$$  \hfill \text{(A-7)}$$

We generate a dichotomous variable $D_1$ based on $F_1$, where $D_1 = 1$ if $F_1 > 0$ and $D_1 = 0$ if $F_1 = 0$.

2.4. The conditional distribution of a father’s variable $U_2$ given $U_1$ and $D_1$ follows a normal distribution, where the standard deviation is fixed at 1 and the mean satisfies the equation below.

$$E(U_2 | U_1, D_1 = 1) = \gamma_1 \cdot (U_1 - \bar{U}_1) = 0.8 \cdot (U_1 - \bar{U}_1)$$  \hfill \text{(A-8)}$$

2.5. The conditional distribution of a father’s position $Y_2$ given $U_2, F_1, D_1$ and $Y_1$ follows a Bernoulli distribution.

$$\logit(P[Y_2 = 2 | U_2, Y_1, D_1 = 1]) = \delta_0 + \delta_1 \cdot U_2 + \delta_2 \cdot Y_1 + \delta_3 \cdot F_1$$

$$= \log \left( \frac{0.2}{0.8} \right) + \log(2) \cdot U_2 + \log(2.5) \cdot Y_1 + \log (1.1) \cdot F_1$$  \hfill \text{(A-9)}$$

2.6. The conditional distribution of a father’s fertility $F_2$ given $Y_2, Y_1, F_1$ and $D_1$ follows a Poisson distribution with the mean of the fertility satisfies the equation below.

$$E(F_2 | Y_2, Y_1, F_1, D_1 = 1)$$

$$= \exp(\theta_0 + \theta_1 \cdot (Y_2 - \bar{Y}_2) + \theta_2 \cdot (Y_1 - \bar{Y}_1) + \theta_3 \cdot (F_1 - \bar{F}_1))$$

$$= \exp(\log(1.1) + 0.4 \cdot (Y_2 - \bar{Y}_2) + 0.2 \cdot (Y_1 - \bar{Y}_1) + 0.1 \cdot (F_1 - \bar{F}_1))$$  \hfill \text{(A-10)}$$

We generate a dichotomous variable $D_2$ based on $F_2$, where $D_2 = 1$ if $F_2 > 0$ and $D_2 = 0$ if $F_2 = 0$. 

$$\logit(P[Y_2 = 2 | U_1, \gamma_1, \delta_0, \delta_1, \delta_2, \delta_3, \theta_0, \theta_1, \theta_2, \theta_3, \theta_4, \bar{Y}_1, \bar{Y}_2, \bar{F}_1, \bar{U}_1, \bar{U}_2, \bar{F}_2]) = \alpha_0 + \alpha_1 \cdot U_1 = \log \left( \frac{0.3}{0.7} \right) + \log(2) \cdot U_1$$  \hfill \text{(A-6)}$$
2.7. The conditional distribution of a son’s variable $U_3$ given $U_2, U_1$ and $D_2$ follows a normal distribution, where the standard deviation is fixed at 1 and the mean satisfies the equation below.

$$E(U_3|U_2, U_1, D_2 = 1) = \pi_1 \cdot (U_2 - \bar{U}_2) + \pi_2 \cdot (U_1 - \bar{U}_1)$$

(A-11)

$$= 0.6 \cdot (U_2 - \bar{U}_2) + 0.2 \cdot (U_1 - \bar{U}_1)$$

Note that when $D_2 = 1$, we must have $D_1 = 1$.

2.8. The conditional distribution of a son’s position $Y_3$ given $U_3, Y_2, U_2, F_2, Y_1, U_1$, and $D_2$ follows a Bernoulli distribution.

$$\text{logit}(P[Y_3 = 2|U_3, Y_2, U_2, F_2, Y_1, U_1, D_2 = 1])$$

$$= \lambda_0 + \lambda_1 \cdot U_3 + \lambda_2 \cdot Y_2 + \lambda_3 \cdot U_2 + \lambda_4 \cdot F_2 + \lambda_5 \cdot Y_1 + \lambda_6 \cdot U_1$$

(A-12)

$$= \log \left( \frac{0.15}{0.85} \right) + \log(1.8) \cdot U_3 + \log(2.0) \cdot Y_2 + \log(1.3) \cdot U_2$$

$$+ \log(1.1) \cdot F_2 + \log(1.5) \cdot Y_1 + \log(1.1) \cdot U_1$$

2.9. The conditional distribution of a son’s fertility $F_3$ given $Y_3, Y_2, Y_1, F_2, F_1$ and $D_2$ follows a Poisson distribution with the mean of the fertility satisfies the equation below.

$$E(F_3|Y_3, Y_2, Y_1, F_2, F_1, D_1 = 1)$$

$$= \exp(\zeta_0 + \zeta_1 \cdot (Y_3 - \bar{Y}_3) + \zeta_2 \cdot (Y_2 - \bar{Y}_2) + \zeta_3 \cdot (Y_1 - \bar{Y}_1) + \zeta_4$$

$$\cdot (F_2 - \bar{F}_2) + \zeta_5 \cdot (F_1 - \bar{F}_1))$$

(A-13)

$$= \exp(\log(1.1) + 0.36 \cdot (Y_3 - \bar{Y}_3) + 0.20 \cdot (Y_2 - \bar{Y}_2) + 0.10$$

$$\cdot (Y_1 - \bar{Y}_1) + 0.10 \cdot (F_2 - \bar{F}_2) + 0.03 \cdot (F_1 - \bar{F}_1))$$